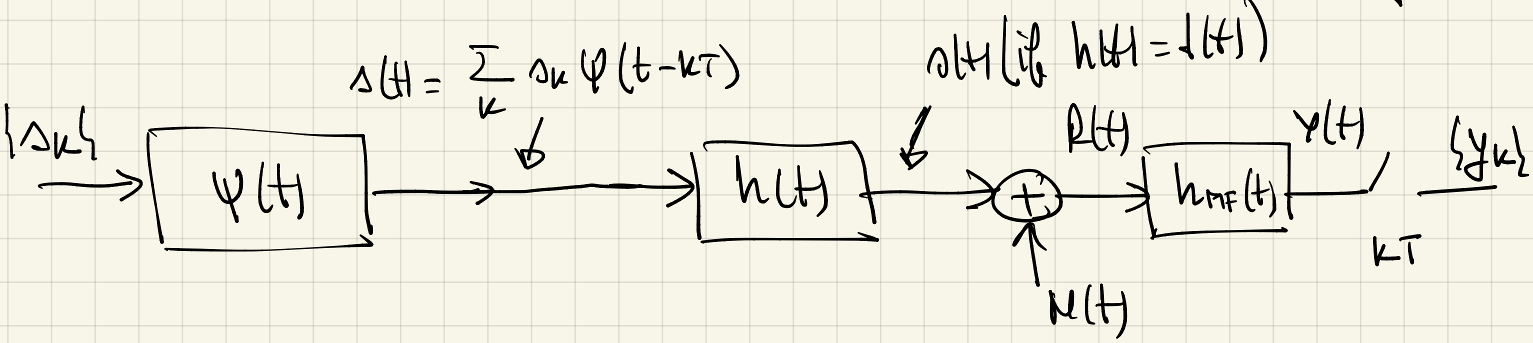
Inter-Symbol Interference (ISI) and the Eye Diagram

Recall the block diagram of our communication system



$\psi(t)$ is a pulse shaping function which satisfies Nyquist criterion, i.e.:

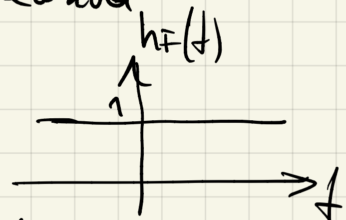
$$\langle \psi(t-kT), \psi(t-jT) \rangle = \delta_{k-j} \quad \begin{cases} 1 & \text{when } k=j \\ 0 & \text{otherwise} \end{cases}$$

A popular choice is the root-raised-cosine (RRC).

$h_{MF}(t) = \psi^*(-t)$

$N(t)$: AWGN, with variance $\frac{N_0}{2}$ per dimension

For now, to fix ideas, assume $h(t) = \delta(t)$



Neglect the noise for now

$$R(t) = s(t) + \cancel{N(t)}$$

$$R(t) = s(t) = \sum_k s_k \psi(t-kT) \qquad \Big\{ \; Y(t) = \underbrace{R(t)}_{s(t)} * h_{MF}(t) = \dots$$

$$Y(t) = (s * h_{MF})(t) = \int s(\tau)\, h_{MF}(t-\tau)\, d\tau =$$

$$= \int \sum_k s_k \psi(\tau-kT)\, \psi^*(\tau-t)\, d\tau =$$

$$= \sum_k a_k \int \psi(\tau - kT)\, \psi^*(\tau - t)\, d\tau \qquad \to \alpha = \tau - t \qquad =$$

$$= \underbrace{\sum_k a_k \int \psi(\alpha + t - kT)\, \psi^*(\alpha)\, d\alpha = y(t)}$$

Define the self-similarity function of ψ

$$R_\psi(t) = \int \psi(\alpha + t)\, \psi^*(\alpha)\, d\alpha = \langle \psi(\alpha+t), \psi(\alpha) \rangle$$

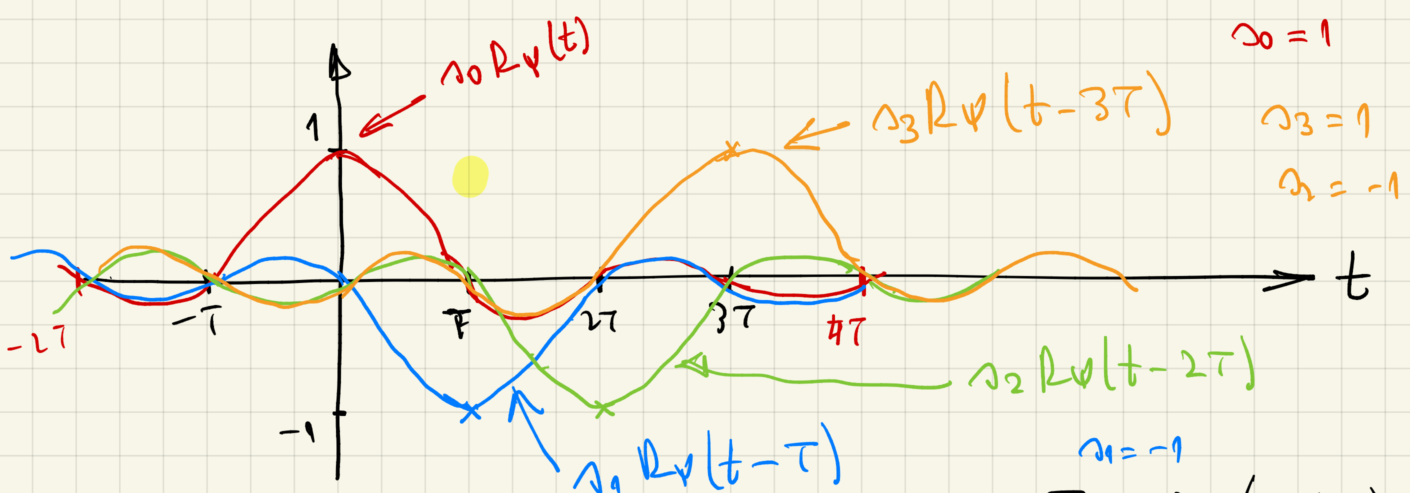
$$\boxed{y(t) = \sum_k a_k R_\psi(t - kT)}$$

Let us compute:

$$y(jT) = \sum_k a_k \underbrace{R_\psi(jT - kT)}_{\substack{1 \text{ when } k=j \\ 0 \text{ otherwise}}} =$$

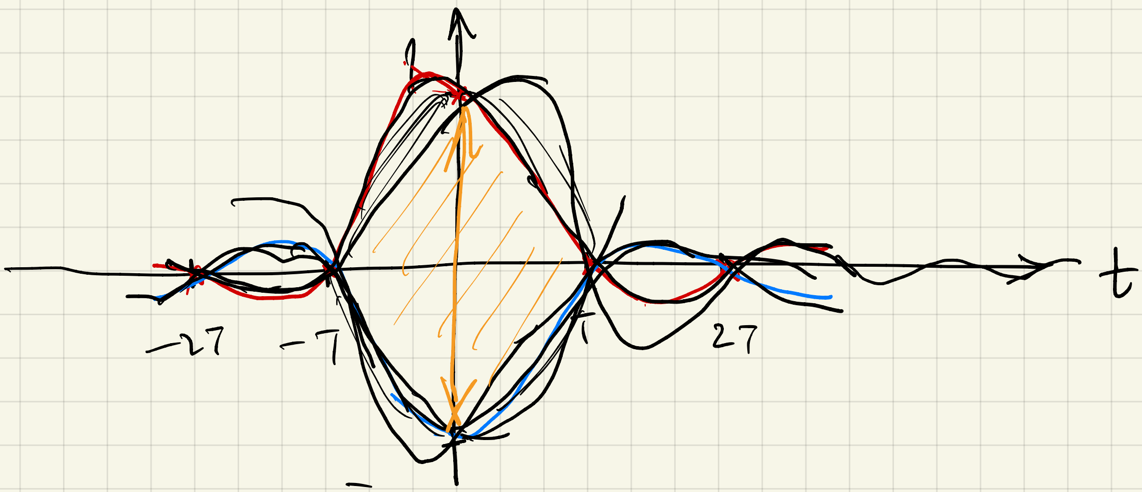
$$= \underline{a_j}.$$

Let assume $a_k \in \{\pm 1\}$, $\psi(t)$ is the NLC.



$$\underline{y(t)} = \sum_k a_k R_\psi(t - kT)$$

How we construct the eye diagram: we plot $y(t - iT)$ for the interval $[-T, T]$ for various is



<u>Our assumptions:</u>

i) $\psi(t)$ is Nyquist $\langle \psi(t-kT), \psi(t-iT) \rangle = \delta_{k-i}$

This is happening if $\psi(t)$ has infinite length

because $\psi_F(f)$ has finite support $[-\frac{1}{T}, \frac{1}{T}]$

ii) $h(t) = \delta(t)$

iii) Assumed we sample exactly at kT (there is no offset)

<u>In reality</u>

i) We have to truncate $\psi(t)$, for practical reasons

$\Rightarrow$ $\varphi(t)$ will not be anymore a Nyquist pulse (the truncated $\varphi(t)$)

ii) $h(t) \neq \delta(t)$ (Frequency selective channel)

iii) You might sample with some offset at $\underbrace{kT + t_0}$ $\rightarrow$ offset